A Pilot Study on Functional Processing: Inferences of Pairwise Relationships in Systems of Three Variables

Kent L. Norman and Benjamin K. Smith

Laboratory for Automation Psychology and Decision Processes
Department of Psychology
University of Maryland
College Park, Maryland 20742-4411

Introduction

The world is full of many variables and relationships among the variables. When people make judgments and decisions, they do so often on the basis of what they believe about the relationships among variables, especially when predicting of one variable on the basis of a set of other variables (e.g., Anderson, 1981; Norman, 1974). For example, with price and quality, we may know that as quality goes up, so will price. From this, we may infer that as price goes up, so does quality, even though it is not necessarily true. This inference is one of bi-directionality; and it is often true for both correlational and deterministic relationships if there are no mitigating forces.

A second type of inference is transitivity. Given a set of three variables, A, B, and C, if B increases with A and C increases with B, then we might infer that C increases with A. Again, this is not necessarily true and one can think of many counterexamples; but in many situations transitivity is appropriate.

Most studies on the relationships of variables and inferences about them have been in context of multivariate relationships such as Brunswik's lens model (Brunswik, 1955) and Hammond's social judgment model (Hammond, Stewart, Brehmer, Steinmann, 1975). In these examples, one variable is deemed the criterion variable and the rest are referred to as cues or predictor variables. But in many situations, there are only sets of variables and imperfect knowledge about their inter-relationships. We may use this partial knowledge concerning the inter-relationship between some variables to infer the relationship between other variables. In a sense, we find ourselves in a task of intuitive statistics dealing with causal reasoning and structural equation modeling and path analysis (Campbell & Stanley, 1966; Byrne, 1994).

In other literatures on reasoning, such as syllogistic logic (e.g; Revlin & Mayer, 1978; Wilkins, 1928), there is a prescriptive truth. If all A are B and all B are C, then it is true that all A are C. But in the relationships among three variables, there is no such prescriptive truth. Inferences may depend on the weight of linguistic tendencies such as with atmosphere theory (Woodworth & Sells, 1935; Chapman & Chapman, 1959) or with the ability to generate instances (e.g., Revlis, 1975). Moreover, with sets of the variables, the instances will depend on the context and its implication on the structure of the system. One would expect different systems of inference to apply for sets of variables in mathematical systems (e.g., A = B + C), physical systems (e.g., A = velocity, B = mass, C = force), economic systems (e.g., A = net profit, B = advertising cost, C = market penetration), ecological systems (e.g., A = population of Species 1, B = population of Species 2, C = population of Species 3), and social and personality systems (e.g., A = self esteem, B = personal income, C = number of friends).

In this pilot study, we initiate a line of research on inference about pairwise relationships in systems of variables. We present a new task and experimental design in which we present two relationships and then ask the participant to infer a third relationship. Each task is of the form:

As A increases, B increases.
As C decreases, B increases.
Then as B increases, what happens to A?

- increases
- decreases
- no change

Each letter (A, B, C) is replaced in the problem by the name of some chemical.

Experiment

Participants.

Twenty-four undergraduates participated in the study as partial fulfillment of a course requirement. Seven were female and 16 were male (one did not indicate gender on the demographics questionnaire). Their ages ranged from 17 to 24 with a mean of 19.75. When asked to give a self rating of use of overall use of computers (1=no experience, 10=very experienced) the mean was 7.13 (s.d. = 1.68). When asked to give a self rating on the World Wide Web on the same scale, the mean was 7.39 (s.d. = 1.37).

Task Materials

Seventy-two judgment problems were generated by varying the relationship between the first two variables and second two variables and permuting the order of the variables. The problems are listed in Table 1. Sets of chemical names were randomly selected from a pool and substituted for the letters A, B, and C for each participant such that no two names in one problem began with the same letter. The order of the 72 problems was supposed to have been randomized for each of the participant; but due to an error in the database, they were randomized once, and presented in the same order for all of the participants.

All problems were presented in a browser window and displayed on a 15 inch flat panel monitor. The task was administered on Apple Macintosh computers running MacOS X 10.2 and the Safari web browser. Responses were indicated by clicking on one of three radio buttons for the inferred relationship. Figure 1 shows a screen shot of one of the problems. After the participant clicked on a relationship and clicked on the "Continue" button, the browser paged to the next problem.

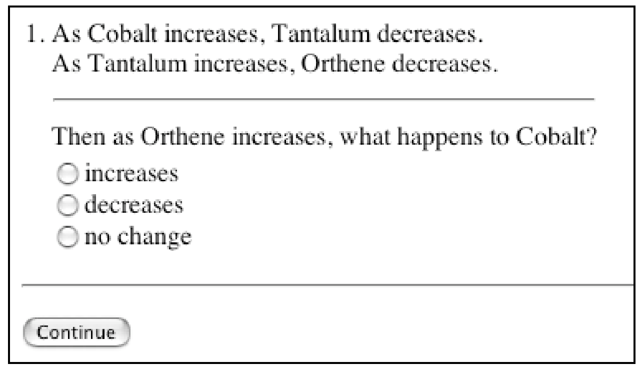

Figure 1. Screen shot of the one of the 72 relationship inference problems.

Procedure

The participants agreed to the conditions of the informed consent and their questions were answered. The participants filled in a pre-test questionnaire for age, gender, and self-report of computer knowledge and use of the World Wide Web. The judgment task was then described to the participants. They were told that they were to help a geologist make inferences about the relationships among chemicals in rocks. They would be told two relationships and then had to infer the third. When they were finished they were asked to take a test of spatial visualization ability called the VZ2 (Ekstrom, French, & Harmon, 1976). This took six minutes and was administered in the browser window. After the test, they were debriefed and asked if they had any questions.

Table 1
Relationship Problems

#	1st Rel.			2nd Rel.			3rd Rel.		Inc.	N. C.	Dec.
1	A	B	+	A	C	+	B	C	11	0	0
2	A	B	+	A	C	+	C	B	11	0	0
3	A	B	+	A	C	-	B	C	0	1	10
4	A	B	+	A	C	-	C	B	0	0	11
5	A	B	+	A	C	0	B	C	0	11	0
6	A	B	+	A	C	0	C	B	0	10	1
7	A	B	+	C	A	+	B	C	10	0	1
8	A	B	+	C	A	+	C	B	10	1	0
9	A	B	+	C	A	-	B	C	1	0	10
10	A	B	+	C	A	-	C	B	0	0	11
11	A	B	+	C	A	0	B	C	2	8	1
12	A	B	+	C	A	0	C	B	1	7	3
13	A	B	-	A	C	+	B	C	1	1	9
14	A	B	-	A	C	+	C	B	0	0	11
15	A	B	-	A	C	-	B	C	10	0	1
16	A	B	-	A	C	-	C	B	10	0	1
17	A	B	-	A	C	0	B	C	2	9	0
18	A	B	-	A	C	0	C	B	2	8	1
19	A	B	-	C	A	+	B	C	0	2	9
20	A	B	-	C	A	+	C	B	1	0	10
21	A	B	-	C	A	-	B	C	7	0	4
22	A	B	-	C	A	-	C	B	10	0	1
23	A	B	-	C	A	0	B	C	1	8	2
24	A	B	-	C	A	0	C	B	0	10	1
25	A	B	0	A	C	+	B	C	0	10	1
26	A	B	0	A	C	+	C	B	0	11	0
27	A	B	0	A	C	-	B	C	1	9	1
28	A	B	0	A	C	-	C	B	1	10	0
29	A	B	0	A	C	0	B	C	5	6	0
30	A	B	0	A	C	0	C	B	4	7	0
31	A	B	0	C	A	+	B	C	1	8	2
32	A	B	0	C	A	+	C	B	1	10	0
33	A	B	0	C	A	-	B	C	2	7	2
34	A	B	0	C	A	-	C	B	0	11	0
35	A	B	0	C	A	0	B	C	3	6	2
36	A	B	0	C	A	0	C	B	1	9	1
37	A	B	+	B	C	+	A	C	10	1	0
38	A	B	+	B	C	+	C	A	11	0	0
39	A	B	+	B	C	-	A	C	1	1	9
40	A	B	+	B	C	-	C	A	0	0	11

41	A	B	+	B	C	0	A	C	0	11	0
42	A	B	+	B	C	0	C	A	3	7	1
43	A	B	+	C	B	+	A	C	11	0	0
44	A	B	+	C	B	+	C	A	11	0	0
45	A	B	+	C	B	-	A	C	0	1	10
46	A	B	+	C	B	-	C	A	0	1	10
47	A	B	+	C	B	0	A	C	1	9	1
48	A	B	+	C	B	0	C	A	1	10	0
49	A	B	-	B	C	+	A	C	0	1	10
50	A	B	-	B	C	+	C	A	3	1	7
51	A	B	-	B	C	-	A	C	10	0	1
52	A	B	-	B	C	-	C	A	7	0	4
53	A	B	-	B	C	0	A	C	1	9	1
54	A	B	-	B	C	0	C	A	0	9	2
55	A	B	-	C	B	+	A	C	1	1	9
56	A	B	-	C	B	+	C	A	2	1	8
57	A	B	-	C	B	-	A	C	10	0	1
58	A	B	-	C	B	-	C	A	7	4	0
59	A	B	-	C	B	0	A	C	3	5	3
60	A	B	-	C	B	0	C	A	1	8	2
61	A	B	0	B	C	+	A	C	2	9	0
62	A	B	0	B	C	+	C	A	1	9	1
63	A	B	0	B	C	-	A	C	0	8	3
64	A	B	0	B	C	-	C	A	1	7	3
65	A	B	0	B	C	0	A	C	0	9	2
66	A	B	0	B	C	0	C	A	3	8	0
67	A	B	0	C	B	+	A	C	3	7	1
68	A	B	0	C	B	+	C	A	2	9	0
69	A	B	0	C	B	-	A	C	3	7	1
70	A	B	0	C	B	-	C	A	1	8	2
71	A	B	0	C	B	0	A	C	7	4	0
72	A	B	0	C	B	0	C	A	9	2	0

= Problem number. Not the presentation order.
1st Rel. = Components, value of 1st relationship
2nd Rel. = Components, value of 2nd relationship
Inc. = Number of “Increases” responses
N. C. = Number of “No Change” responses
Dec. = Number of “Decreases” responses

Results

Due to response omissions, half of the 22 participants had between one and five missing data points. The missing responses comprised 1.5% of all of the data. Consequently, the data were analyzed two different ways. First, all 22 participants were included, and missing data points were treated as "no change" responses. This introduces a slight bias in favor of "no change," but does not introduce bias for or against "increases" or "decreases." Second, only the 11 subjects with no missing data points were considered. This sample had no bias, but less power. Both samples were tested using a repeated-measures ANOVA. The factors of the ANOVA were the "+-0" condition, the "ABC" condition, and the interaction. The "+-0" condition indicated the value of the first two relationships in the question. The "ABC" condition indicated the positions of the element names in the second and third parts of the question. The effects for both conditions and the interaction for both samples were significant,
($p < .05$). The "+-0" effect size was larger than the “ABC” effect.

Although the missing data responses were counted as "no change" responses for the repeated-measures ANOVA, it is easier to compare the samples by simply excluding participants with the missing data. For the following comparisons, the 22-subject sample simply had the missing data points removed. For the sake of simplicity, all statistics quoted below and the values in Table 1 are from the unbiased 11-subject sample. Where the 22-subject sample differs more than a few points, this will be noted. All percentages are rounded to the nearest 1%.

Let us first look at the "+-0" factor. Each condition will be indicated with two signs, the first indicating the value of the first relationship, and the second, the value of the second relationship. (The response was the participant’s guess as to the value of the third relationship.) For instance, +- is the condition where the first relationship is "increases," and the second, "decreases." A "no change" is indicated by a 0.

If the first two relationships had the same non-zero value, the responses were overwhelmingly "increases." The "++" condition had 97% "increases" responses, and the "--" condition had 81% "increases" responses.

If the first two relationships had different non-zero value, the responses were overwhelmingly "decreases." The "+-" condition had 93% "decreases" responses, and the "-+" condition had 83% "decreases" responses.

If one of the first two relationships was "doesn't change," the majority of responses were "doesn't change," with the remaining responses split, but always favoring the same value as the non-zero relationship. Condition "+0" had 83% "doesn't change," "-0" had 75%, "0+" had 83%, and "0-" had 76%. The largest imbalance was for "0+", which had 11% "increases," and 6% "decreases." The largest imbalance favoring decreases was "-0", 14% compared to 11% increases.

Finally, when the first two relationships were both "doesn't change," the majority of responses were "doesn't change," but only 58% (just 53% in the full sample.) A large minority of responses (36%) were "increases." This was by far the condition with the weakest dominant answer as shown in Figure 2.

The differences among the "ABC" conditions were not as large, but were also significant. These conditions are indicated by two pairs of two letters. They represent the order of the chemical names in the second and third parts of the question. The first two chemicals are always "A" and "B." The question asks about the value of the relationship between the last pair of chemicals. If the question first states a relationship between "A" and "B," then between "A" and "C," and then asks the value of the relationship between "B" and "C," this would be indicated as "AC BC."

For each response, the most extreme conditions are listed.

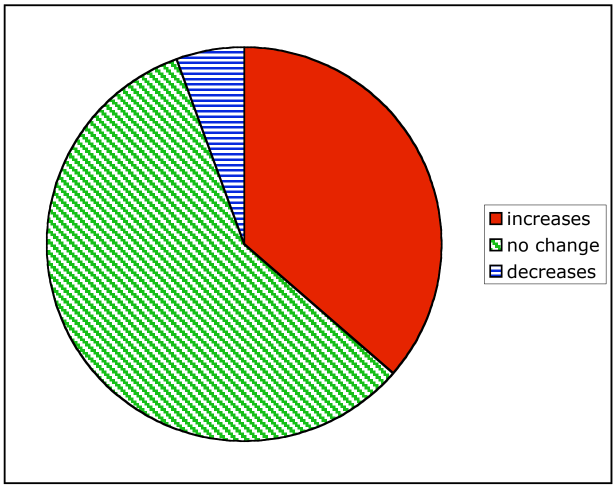

Figure 2. Proportions of participants inferring each type of relationship for the “Doesn’t change, Doesn’t change” problem.

In the "CB AC" condition, 39% of responses were "increases" as shown in Figure 3. This was even higher, 43%, in the full sample. For the "BC AC" and "CA CB" conditions, the percentage of "increases" responses was 24%. These were also somewhat higher in the full sample.

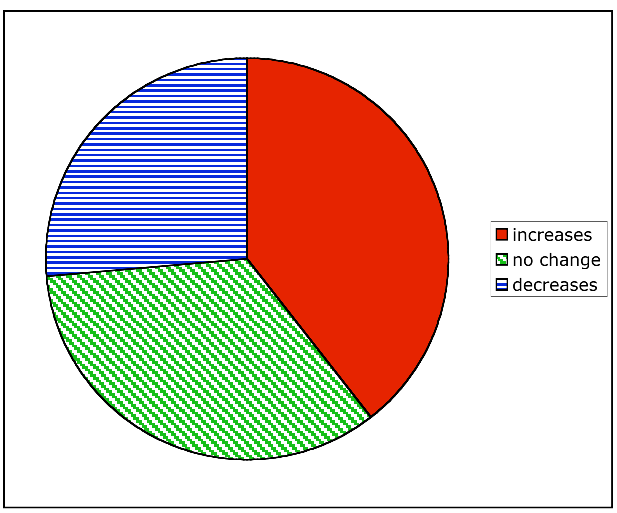

Figure 3. Proportions of participants inferring each type of relationship for the “CD AC” problem.

In the "CA BC" condition, 33% of the responses were "decreases." The "AC BC" and "CB CA" conditions had only 22% "decreases" responses.

In the "BC AC" condition, just under 50% of the responses were "doesn’t change." The "CB AC" condition had 34% "doesn’t change" responses.

Finally, the average VZ2 score for the participants was 11.33 (s.d. = 4.16). We had planned a median split on the sample of 22 to generate a Hi VZ2 Group and a Lo VZ2 Group; however, due to a coding error, it was impossible to identify which scores went with which participants.

Discussion

The results are encouraging in that clear patterns were evident in the data indicating that participants were not merely responding randomly but were making consistent inferences on the basis of some principles or strategies. Figure 4 shows a diagram of the predominate patterns of inference.

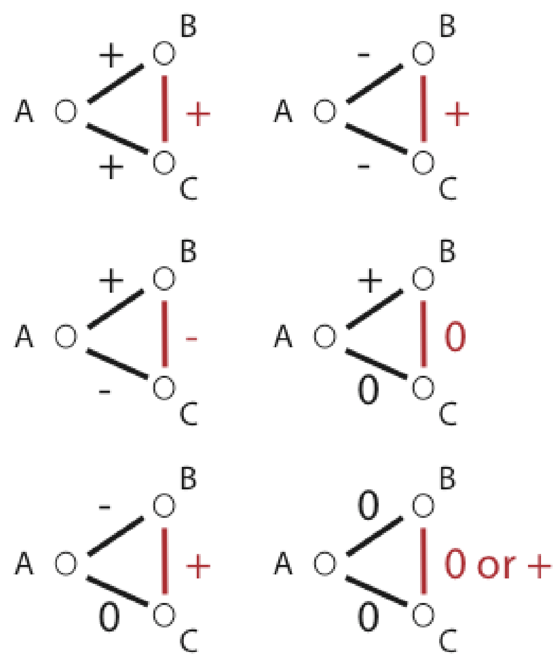

Figure 4. Diagrams of the predominate patterns of inference for the six relational structures. Red arrows show the inferred relationship.

Clearly, the most dominant pattern was for positive relationships (e.g., As A increases, B increases; and as B increases, C increases.) to lead to a positive inference (e.g., As A increases, C increases). Moreover, the order could be changed and participants still made the same positive inference suggesting that they were inferring bi-directional and transitive relationships.

The second clear pattern was for two negative relationships to result in a positive relationship between the third pair. The inference is that if both are in the same negative direction, the two variables must be moving in the same (positive) direction with each other.

Mixed relationships on the other hand result in the inference of a negative relationship. This is in line with the inference above. If one does one way and the other in the opposite direction, then the two variables must be negatively related.

When there is a “doesn’t change” for one of the relationships, it nullifies the effect of the other (whether positive or negative) and results in the inference of no relationship between the two variables.

Finally, if both relationships are “doesn’t change,” there is a split with the majority inferring that there is no relationship, but with a substantial minority inferring a positive relationship between the two variables. It is not clear at this point, what the basis is for this inference.

Limitations

Several problems in the design of this pilot study became evident. The first two problems had to do so with technical software issues. First, the order of the 72 items should have been randomized for each participant to eliminate order effects. Second, the design of the software allowed the participants to omit or skip responses. This resulted in a loss of data and resulting problems with the statistical analysis. These two problems will be corrected in subsequent experiments.

The third problem was that participants were forced to answer with only three options: increase, decrease, and no change. They were not allowed to say that they did not know. This may have forced them to guess in situations where they were not able to make an inference. On the other hand, if they are allowed to respond, “don’t know”, they may over use this option. An experiment is needed to test what will happen with the presence or absence of the “don’t know” option.

Fourth, only one context or scenario was tested in this study; namely, the chemical composition of rocks. Subsequent experiments should vary the type of system of variables to see if inferences change.

Finally, in this study we used the VZ2 test of spatial visualization ability. In subsequent research, it might be preferable to use a test of formal reasoning such as a test of syllogistic reasoning.

Conclusion

If relationships between variables are inferred with some consistency from other relationships among variables in a set, these inferences can be used to make judgments and predictions. In many judgment situations, the most important thing is to know the direction of the relationship between the cue and the criterion (e.g., Dawes). These directions may come from inferences using information about other relationships in the set of variables.

This study is an initial attempt to understand these inferences and serves as the starting point for the development of a theory of functional processing.

Acknowledgments

Thanks to Robert Rowsome and Melissa Tortoriello. This work as funded in part from a grant from the U.S. Census Bureau, Statistical Research Division, Grant 50YABC166008.

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